Enriched Serial-Loop Optimization Method for Efficient Reliability-Based Electromagnetic Designs

Dong-Wook Kim[1], Byungsu Kang[1], K. K. Choi[2] and Dong-Hun Kim[3]

[1]Dept. of Electrical Eng., Kyungpook National Univ., Daegu 702-701, Republic of Korea, bloodkdw@nate.com
[2]Dept. of Mechanical and Industrial Eng., Univ. of Iowa, Iowa City, IA 52242-1527, USA, kyung-choi@uiowa.edu
[3]Dept. of Electrical Eng., Kyungpook National Univ., Daegu 702-701, Republic of Korea, dh29kim@ee.knu.ac.kr

An enriched serial-loop method for reliability-based design optimization is presented to substantially enhance computational efficiency as well as numerical accuracy when applied to electromagnetic design problems in the presence of uncertainties. In the method, two improvements are made over the original serial-loop optimization method, which employs a serial of cycles of equivalent deterministic optimization and reliability analysis. One is a feasibility check technique for probabilistic constraints conducted only after the first design cycle. It can identify inactive constraints, which need not be taken into account during the next design cycles. The other is a reliability improvement scheme for slightly violated probabilistic constraints performed at the end of iterative design cycles. When an optimum point is unsatisfactory to all probabilistic constraint conditions given, the design point is shifted toward a feasible design region by utilizing probabilistic information obtained. Finally, the TEAM Workshop Problem 22 is tested to compare the proposed method with the original one from a numerical efficiency and accuracy point of view.

***Index Terms*—Electromagnetics, optimization, reliability theory, robustness.**

I. Introduction

In recent years, a probabilistic optimization called reliability-based design optimization (RBDO) has been introduced to systematically incorporate uncertainties involved in electromagnetic (EM) system inputs into an early design stage [1]-[6]. From the viewpoint of program architecture, the RBDO can be categorized into two different methods: parallel-loop and serial-loop. In the parallel-loop RBDO such as performance measure approach (PMA) and sampling-based approach [4]-[5], two optimization loops are interactively executed at each iterative design: to find an optimum of the objective function as a main loop, and to satisfy the given probabilities of constraints as a sub-loop. It is revealed that such parallel-loop RBDO inevitably requires a significant computational burden. In contrast, the serial-loop method fully decouples reliability assessment from the main optimization loop [6]. To verify constraint feasibility under uncertainty, the reliability analysis is only conducted after deterministic optimization with shifting constraint boundaries. This sequential optimization methodology can significantly save a computation cost by decreasing the number of reliability analyses. Although the method seems promising, its computational burden is still much heavier than that of deterministic optimization.

II. Proposed Enriched Serial-Loop RBDO

The original serial-loop optimization model is formulated as follows [6]:

$$\begin{aligned} &\text{minimize} && f(\mathbf{d}) \\ &\text{subject to} && g_i(\mathbf{d}-\mathbf{s}^{k-1}) \le 0, \quad i=1,2,\ldots nc \qquad (1) \\ & && \mathbf{s}^{k-1}=\mathbf{d}^{k-1}-\mathbf{x}_{\mathrm{MPP}}^{k-1} \qquad (\mathbf{s}^0=0) \end{aligned}$$

where f is the objective function, $\mathbf{d}$ is the design variable vector defined by $\mathbf{d}=\boldsymbol{\mu}(\mathbf{X})$, where $\boldsymbol{\mu}$ denotes the mean of random variables $\mathbf{X}$ in X-space, and g_i is the ith function of nc constraints. After deterministic optimization at the kth design cycle, the most probable failure point (MPP), $\mathbf{u}_{\mathrm{MPP}}^{k-1}$, in a standardized normal space (U-space) is sought by the reliability assessment of PMA. Then, it is transformed back to X-space for obtaining the inverse MPP, $\mathbf{x}_{\mathrm{MPP}}^{k-1}$. Therefore, $\mathbf{s}^{k-1}$ is a movement direction vector corresponding to the displacement from $\mathbf{x}_{\mathrm{MPP}}^{k-1}$ to a deterministic optimum, $\mathbf{d}^{k-1}$, at the previous design cycle (k-1). Instead of dealing with probabilistic constraints directly, the original method consecutively executes a serial design of deterministic optimization with shifting constraints and reliability analysis.

However, although a wealth of information is generated through the reliability assessment at each cycle, it is not reused in the original method. This paper thus focuses on developing a more efficient and accurate serial-loop optimization method through introducing two numerical techniques consisting of a feasibility check for probabilistic constraints and a reliability improvement scheme for slightly violated constraints.

A. *Feasibility Check for Probabilistic Constraints*

Based on the probabilistic information obtained after the first design cycle, the feasibility identification is performed. The foreseeable feasibility status of probabilistic constraints is illustrated in Fig.1. Three different locations of a deterministic optimum $\mathbf{d}^1$ relative to the constraint g_i are assumed. Thereby, the feasibility status is classified as follows:

a) Inactive probabilistic constraint (Case A). A constraint is considered to be feasible at the design point if $g_i(\mathbf{x}_{\mathrm{MPP}}^1) < -\varepsilon$, ε is a small positive number.

b) Active probabilistic constraint (Case B). Any constraint is ε active at the design point if $-\varepsilon \le g_i(\mathbf{x}_{\mathrm{MPP}}^1) \le 0$.

c) Violated probabilistic constraint (Case C). A constraint is said to be violated at the design point if $g_i(\mathbf{x}_{\mathrm{MPP}}^1) > 0$.

After all, only two potential probabilistic constraints (active and violated ones) are involved from the second design cycle.

It can lead to improving the numerical efficiency of the original serial-loop optimization method.

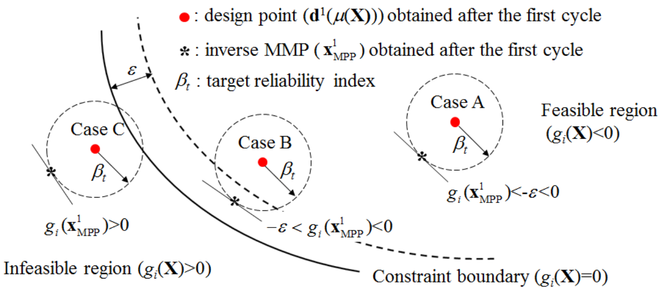

Fig. 1. Feasibility status of probabilistic constraints.

B. *Reliability Improvement Scheme*

Due to the limitation of numerical simulation, the convergence criterion for probabilistic constraints is generally defined by using a tolerance value ε_f. If $|g_i(\mathbf{x}^k_{\mathrm{MPP}})| \le \varepsilon_f$ from i=1 to nc at the kth cycle, probabilistic constraint conditions given is assumed to be satisfied. In the case of $0< g_i(\mathbf{x}^k_{\mathrm{MPP}}) \le \varepsilon_f$, however, the ith constraint condition is still violated at a final design point $\mathbf{d}^*$ even though its reliability value is very close to a target one. In this case, $\mathbf{d}^*$ is forced to move toward a feasible design region by $\varepsilon_f\, \mathbf{s}^{k\text{-}1}/|\,\mathbf{s}^{k\text{-}1}|$ without any additional reliability analysis. Such a reliability improvement scheme can result in satisfying all probabilistic constraint conditions.

The flowchart of the enriched serial-loop design strategy for RBDO is provided in Fig. 2, where two dotted boxes are newly added in the original one.

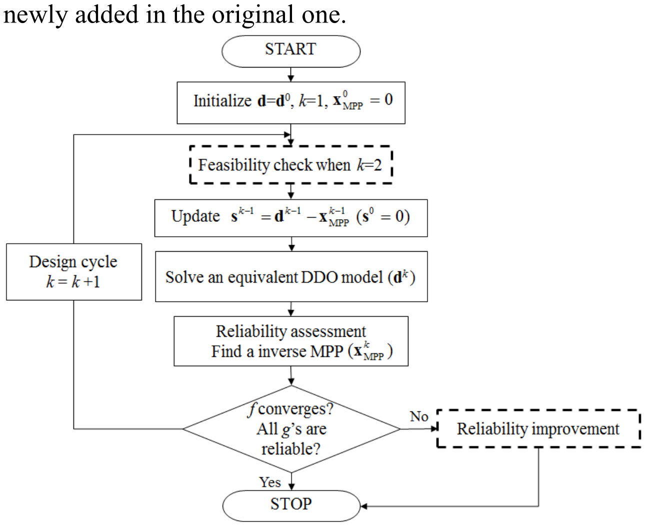

Fig. 2. Flowchart of the proposed serial-loop strategy for RBDO.

III. Results

Taking into account the presence of uncertainty, the original eight-parameter problem of the TEAM benchmark problem 22 presented in [3] and [6] was tested. Its conventional RBDO formulation is written by

$$\begin{aligned} &\text{minimize} \;\; f(\mathbf{d}) = B^2_{stray}(\mathbf{d})/B^2_{norm} + \left|E(\mathbf{d})-E_0\right|/E_0 \\ &\text{subject to} \quad P_F(g_i(\mathbf{X})>0) \le P_{t,i} \quad i=1,2,3 \\ &g_1(\mathbf{X}) = (R_2-R_1) - \frac{1}{2}(D_2+D_1) \\ &g_{2,3}(\mathbf{X}) = -\left|\mathrm{J}_k\right| - 6.4\left|\mathrm{B}_{\max,k}\right| + 54.0 \quad k=1,2 \end{aligned} \tag{2}$$

where B_{stray} is the stray field, B_{norm} is 200 μT, and E is the stored magnetic energy of a superconducting magnetic energy storage system relative to the target value E_o of 180 MJ. The target probability of failure $P_{t,i}$ is set to be 5% for all constraints (i.e. reliability of 95%). It is assumed that all eight random parameters comply with normal probabilistic distributions, of which standard deviation (SD) values are presented in Table I. Electromagnetic field simulations were conducted with a commercial finite element analysis (FEA) code, called MagNet.

Starting with a deterministic design optimization (DDO) point, the serial-loop formulation of (2) was solved with the original and the proposed methods. Table I shows performance indicators between three different design points. It is observed that the DDO violates two probabilistic constraint conditions of g_2 and g_3, and the original RBDO slightly violates the probabilistic condition of g_3. Whereas the proposed RBDO satisfies all probabilistic conditions. Specifically, the FEA calls of the proposed method is smaller by more than 60% than that of the original one. It is inferred that the proposed method significantly enhances computational efficiency as well as numerical accuracy of RBDO.

More detailed explanation and comparative results will be presented in the extended version of the paper.

TABLE I
Design Variables and Performances at Three Different Designs

Design variables	Unit	d^L	SD	d^U	DDO	Serial-loop RBDO Original	Serial-loop RBDO Proposed
R_1	mm	1000	10	2000	1296	1313	1313
D_1	mm	100	6	800	583	634	633
$H_1/2$	mm	1000	10	1800	1089	1141	1139
R_2	mm	1000	10	2000	1800	1832	1831
D_2	mm	100	2	800	195	189	188
$H_2/2$	mm	1000	10	1800	1513	1524	1523
J_1	A/mm^2	10.00	0.01	30.00	16.70	14.95	14.98
J_2	A/mm^2	-30.00	0.01	-10.00	-18.91	-19.72	-19.71
B_{stray}	μT	-	-	-	15.8	46.4	52.5
E	MJ	-	-	-	179	180	180
$P_F(g_1)$	%	-	-	-	0	0	0
$P_F(g_2)$	%	-	-	-	67.29	0	0
$P_F(g_3)$	%	-	-	-	5.35	5.02	4.60
Iterative designs	-	-	-	-	-	18(3*)	18(3*)
FEA calls	-	-	-	-	-	1314	492

* P_F was recalculated by Monte Carlo simulation with 500,000 samples and the number in parenthesis denotes the number of design cycles.

IV. References

[1] A. Haldar and S. Mahadevan, *Probability, reliability, and statistical methods in engineering design*, John Wiley & Sons, New York/Chichester, UK, 2000.

[2] F. Guimaraes, *et al.*, "Multiobjective approaches for robust electromagnetic design," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 1207-1210, Apr. 2006.

[3] G. Soares, *et al.*, "Robust multi-objective TEAM 22 problem: A case study of uncertainties in design optimization," *IEEE Trans. Magn.*, vol. 45, pp. 1028-1031, 2009.

[4] D. Kim, et al., "Optimization of a SMES magnet in the presence of uncertainty utilizing sampling-based reliability analysis," *J. Magn.*, vol. 19, no. 1, pp. 78-83, Mar. 2014.

[5] D. Kim, et al., "Composite first-order reliability method for efficient reliability-based optimization of electromagnetic design problems, *IEEE Trans. Magn.*, vol. 50, no. 2, pp. 7016804, 2014.

[6] D. Kim, et al., "A single-loop strategy for efficient reliability-based electromagnetic design optimization," *IEEE Trans. Magn.,* in press.